AP Calculus AB
Mixed Integration Practice

Name Key

1. A particle's velocity is given in ft/sec by the equation $v(t) = 6t^2 - 6t - 12$, find:
   a) the displacement of the particle in the first 4 seconds
   b) the total distance traveled by the particle on the same interval as in part a

a) $\int_0^4 6t^2-6t-12\,dt$

$2t^3-3t^2-12t\Big|_0^4$

$[2(4)^3-3(4)^2-12(4)]-[0]$

$128-48-48$

32 ft.

b) $0=6t^2-6t-12$

$0=6(t^2-t-2)$

$0=6(t-2)(t+1)$

$t=2$ sec $\quad t=$ ~~-1~~ sec

$\left|\int_0^2 6t^2-6t-12\,dt\right| + \int_2^4 6t^2-6t-12\,dt$

$2t^3-3t^2-12t\Big|_0^2 \qquad 2t^3-3t^2-12t\Big|_2^4$

$[2(2)^3-3(2)^2-12(2)]-[0] \qquad [32]+[+20]$

$16-12-24$

$|-20| + 52 = 72$ ft.

2. Approximate $\int_0^4 x^2-3\,dx$ using:
   a) left endpoints with 8 sub-intervals
   b) right endpoints with 8 sub-intervals
   c) midpoint rule with 8 sub-intervals
   d) trapezoid rule using 8 sub-intervals

$\Delta x = \frac{4-0}{8} = \frac{1}{2}$

$x_0=0 \quad x_1=.5 \quad x_2=1 \quad x_3=1.5 \quad x_4=2$

$x_5=2.5 \quad x_6=3 \quad x_7=3.5 \quad x_8=4$

a) $(.5)[f(0)+f(.5)+\cdots+f(3.5)] = 5.5$

b) $(.5)[f(.5)+f(1)+\cdots+f(4)] = 13.5$

c) $(.5)[f(.25)+f(.75)+\cdots+f(3.75)] = 9.25$

d) $\left(\frac{.5}{2}\right)[f(0)+2f(.5)+2f(1)+\cdots+2f(3.5)+f(4)] = 9.5$

actual area = 9.333

Evaluate each definite integral by hand:

3. $\int_1^2 \frac{x^2+3}{x}\,dx$

$\int_1^2 (x^2+3)x^{-1}\,dx$

$\int_1^2 x+3x^{-1}\,dx$

$\frac{1}{2}x^2+3\ln|x|\Big|_1^2$

$\left[\frac{1}{2}(2)^2+3\ln 2\right]-\left[\frac{1}{2}(1)^2+3\ln(1)\right]$

$\frac{3}{2}+3\ln 2$

4. $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

$\tan x\Big|_0^{\frac{\pi}{4}}$

$\tan\left(\frac{\pi}{4}\right)-\tan(0)$

$1-0$

1

Evaluate each indefinite integral:

5. $\int 2x^3+\sec x-\frac{3}{x^2}\,dx$

$\frac{1}{2}x^4+\ln|\sec x+\tan x|+3x^{-1}+C$

6. $\int \frac{4}{x}+\csc x\cot x+7\,dx$

$4\ln|x|-\csc x+7x+C$

7. $\int \frac{x^4-3x^2-5}{\sqrt{x}}\,dx$

$\int (x^4-3x^2-5)x^{-1/2}\,dx$

$\int x^{7/2}-3x^{3/2}-5x^{-1/2}\,dx$

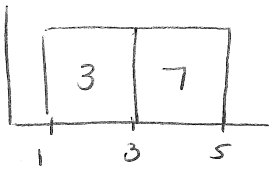
$\frac{2}{9}x^{9/2}-\frac{2}{5}x^{5/2}-\frac{10}{3}x^{3/2}+C$

8. $\int 4e^x+\tan(x)-\sec^2(x)\,dx$

$4e^x-\ln|\cos x|-\tan x+C$

9. Given: $\int_3^1 f(x)dx=-3$ & $\int_3^5 f(x)dx=7$

Find: $\int_1^5 f(x)dx$ $= 10$



10. Given: $\int_0^9 f(x)dx=12$ & $\int_9^3 f(x)dx=5$

Find: $\int_0^3 f(x)dx$ $= 17$